Indian J. Phys. **75B** (6), 533–537 (2001)

I J P B
— an international journal

Laser cooling of two-level atoms by two-photon transition

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Received 29 December 2000, accepted 3 October 2001

Abstract It is shown that nonlinear interaction between two-level atom and the radiation field can profoundly modify the optical force experienced by the atom The dynamic stark shift now provides a cooling force even at zero detuning. Due to two-photon interaction the 'radiation pressure force' experienced by the atom is twice as large as that for one-photon case, consequently temperatures much lower than the one-photon limit can be achieved

Keywords Two-photon interaction, laser cooling, optical force

PACS Nos. 32 80 Pj, 42 50 Vk

1. Introduction

Recent advances in laser cooling of atoms have been attracting considerable attention during the past few years [1–6]. Fundamental results demonstrate that it should be possible to probe concepts developed for condensed matter at totally different scales using optical lattices, where the defect free confining potential is created by light interference [7–9]. The realization of optical traps was made possible by techniques to laser cool neutral atoms to the temperature in the microkelvin range. A slowly moving atom in a low intensity standing laser light wave experiences a velocity dependent force. This 'radiation pressure' force can cool the atom for laser detunings to the red side of the atomic transition or heat the atom for blue detunings. At high intensities, the force changes sign and the effect is reversed. This cooling mechanism has been explained within the frame work of the dressed-atom model [10] and equivalently as a result of photon recoil [11]. The recoil effect is small when a single photon is absorbed but can be effectively used for cooling of atoms by the cumulative effect of many absorbed photons, in a time long compared to the optical pumping cycle. Nonlinear interaction (*i.e.* two photon absorption and emission) between two level atoms and the radiation field has been shown to give rise to many interesting features. In view of the close connection between the number of photons absorbed and the cooling produced by photon recoil, effort has been made to examine the case of laser cooling of a two level atom by two photon transition In the following, the physical origin of the optical force on a two level atom due to two photon transition is described and then the model is presented followed by the method for calculating the force and then the modified force is compared to the force due to one photon transition. Finally, temperature limits for the two-photon case are discussed.

2. Calculation of force in two-photon scheme

We assume a two-level atom moving with a velocity v interacting with two laser beams and capable of going to the excited state by two photon absorption. In the reference frame of the moving atom the frequencies $(\omega_1 + \omega_2)$ of the two electromagnetic waves with wave vectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ are Doppler shifted to

$$(\omega_1' + \omega_2') = (\omega_1 + \omega_2) - \boldsymbol{v}.(\boldsymbol{k}_1 + \boldsymbol{k}_2). \tag{1}$$

As a result, a transfer of recoil momentum $\Delta p = \pm \hbar(\boldsymbol{k}_1 + \boldsymbol{k}_2)$ takes place between the atom and the two fields.

Where + sign refers to absorption of photon and the − sign to emission of photon. If the absorption is followed by spontaneous emission, there is a net momentum transfer to the atom as the spontaneous emission in arbitrary direction gives no average contribution to the momentum As the atom experiences a momentum recoil upon each radiative event, the absorption force of the atom is given by

$$F_a = \Gamma \rho_{aa}\, h(k_1 + k_2), \tag{2}$$

where Γ is the spontaneous emission rate and ρ_{aa} is the upper level population. The total change in kinetic energy is

$$(\Delta E)_{em} - (\Delta E)_{ab} = - h\nu.(k_1' + k_2') - h\nu.(k_1' + k_2'), \tag{3}$$

where ' refers to emission process When the atom absorbs (emits) two photons from (into) the two counter-propagating waves of equal frequencies $[(k_1 - k_2)$ and $(k_1' = -k_2')]$, the doppler shift and the absorptive force vanishes and the total energy change is zero. If the atom absorbs two photons from one wave and emits two photons into the counter-propagating wave, it then acquires a momentum kick of $4hk$, twice as large as that due to one photon case.

We now refer to the 'two-photon-two-level' model introduced by Sargent *et al* [12], in which the transition between levels 'a' and 'b' is nearly resonant, but is not dipole allowed (Figure 1). The transitions from 'a' and 'b' to the intermediate state j are dipole allowed but are sufficiently far

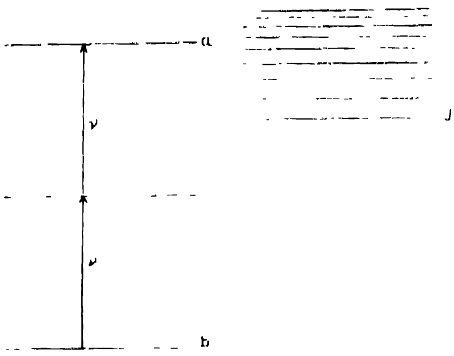

Figure 1. Two-photon two-level scheme

from resonance so that they acquire no appreciable population. The dynamic stark shifts of the level frequencies that ignored in the one-photon case now plays an important role in the two photon case. The optical Bloch equation (OBE) for a two photon two level atom are :

$$\dot{\sigma} = -\gamma\sigma + gD, \tag{4a}$$

$$\dot{D} = \Gamma_1(-D+1) - 2[g\sigma^* + g^*\sigma], \tag{4b}$$

and the force equation is given by

$$F = -ih[\sigma^*\nabla g - \sigma\nabla g^*], \tag{5}$$

where $\gamma = \Gamma_2 + i\Delta'$, $\Delta' = \omega_0 - 2\omega + \omega_s I$, $g = \dfrac{ik_{ab}E^2}{4h}$

$$I = \frac{k_{ab}E^2}{2h\sqrt{\Gamma_1\Gamma_2}}, \omega_s = \quad 2|k_{ab}$$

ω_s is the stark shift parameter, I is the two photon dimensionless intensity, g is the two photon Rabi frequency, k_{ab}, k_{aa}, k_{bb} are the two photon coefficients defined in reference [12], $1/\Gamma_1$ is the population difference (D) decay time and $1/\Gamma_2$ is the two-photon coherence decay time. We take $\Gamma_1 = 2\Gamma_2 = \Gamma$.

In order to calculate the force acting on a slowly moving atom in a standing wave ($k.v < \Gamma$), the semiclassical treatment of Gorden and Ashkin [13] is adopted. If we represent

$$\nabla g = (\alpha + i\beta)g, \tag{6}$$

where α and β are real, then the force equation expands to

$$F = \alpha ih(g^*\sigma - g\sigma^*) + h\beta(g^*.\sigma + g\sigma^*). \tag{7}$$

Now, a moving atom under the influence of a monochromatic field $E(x,t)$ experiences a modified field, since

$$\frac{dE(x,t)}{dt} = \frac{\partial E(x,t)}{\partial t} + (v.\nabla)E(x,t). \tag{8}$$

Consequently, $\dot{g} = v.(\alpha + i\beta)g$. This modifies the solution of eqs. (4a) and (4b). We can obtain an expression for the force accurate to first order in the velocity by taking the time derivative of the zero-order solutions of eqs. (4a) and (4b), using $\dot{g} = v.(\alpha + i\beta)g$ for an atom moving with a velocity v and then using these first order results for σ and $\dot{D}$ to resolve eq. (4) to first order in v. We find thus

$$\dot{D} = -\frac{2p}{(1+p)}(v.\alpha)D, \tag{9a}$$

$$\dot{\sigma} = \left[(v.\sigma)\frac{1-p}{1+p} + i(v.\beta)\right] \tag{9b}$$

where $p = \dfrac{2|g|^2}{|\gamma|^2} \quad \dfrac{I^2}{1 + 4\dfrac{\Delta'^2}{\Gamma^2}}$ is the modified saturation parameter. In this paper, we are interested in the case of a pure standing wave. For this case, we have $\beta = 0$ and for the two-photon case, $\alpha = -2k\tan(2kx)$. We find after some algebra, the following expression of the force due to two-photon interaction.

$$\frac{\alpha h\Lambda' p}{(1+p)}\left\{1 - \frac{[\Gamma^2(1-p) - 2p^2|\gamma|^2]v.\alpha}{\Gamma(1+p)^2|\gamma|^2}\right. \tag{10}$$

It is instructive to examine the expression of the force for one-photon absorption [eq. (18) of Ref. 13]. The saturation parameter for the two-photon case is now dependent on the square of the laser intensity as compared to that for the one-photon case, where it is simply proportional to the intensity

Another important modification of the force in the two-photon case is the term $\Delta' = \omega_0 - 2\omega + \omega_s I$. This term gives rise to a force at zero detuning due to the stark shift parameter. The parameter α is now twice as large as that for one-photon case since the Rabi frequency (g) is now proportional to the square of the electric field. Figure 2 demonstrates the difference of the intensity dependence of the spatially averaged force for one-photon case (trace A) and two-photon case (trace B), showing that the force continues to cool the atom with laser intensity for two photon absorption while it saturates and changes sign to heat the atom in one-photon absorption. Figure 3 demonstrates the dependence of the force on the detuning parameter. Trace A shows that two-

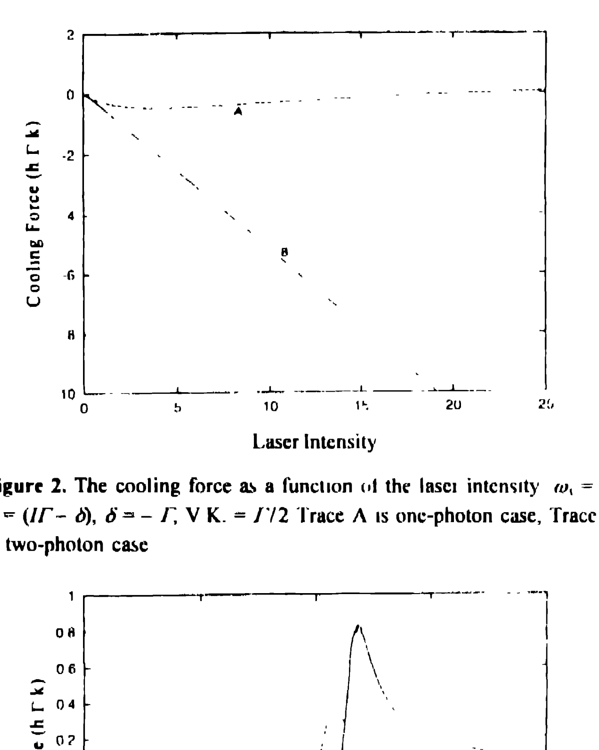

Figure 2. The cooling force as a function of the laser intensity $\omega_s = \Gamma$, $\Delta = (I\Gamma - \delta)$, $\delta = -\Gamma$, V K. $= \Gamma/2$ Trace A is one-photon case, Trace B is two-photon case

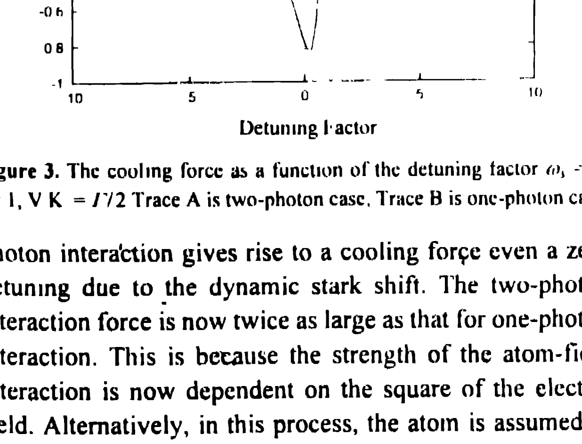

Figure 3. The cooling force as a function of the detuning factor $\omega_s = \Gamma$, $I = 1$, V K $= \Gamma/2$ Trace A is two-photon case, Trace B is one-photon case

photon interaction gives rise to a cooling force even a zero detuning due to the dynamic stark shift. The two-photon interaction force is now twice as large as that for one-photon interaction. This is because the strength of the atom-field interaction is now dependent on the square of the electric field. Alternatively, in this process, the atom is assumed to absorb two photons from one wave and emit two photons into the counter-propagating wave, thus acquiring a momentum kick twice as large as that for one-photon case If the two-photons are absorbed out of the two light waves travelling in the opposite directions, then the first order doppler shift of the two-photon transition is zero. This means that in eq. (2) the force is independent of the atomic velocity and for $\Delta' > 0$ generates a stable trap for an atom sitting on a gaussian beam by pulling it toward the beam focus. Since the probability for the latter case is twice as large as for the former case [11]. It is then suggested that to cool two-level atom *via* two-photon transition, the process should be repeated a number of times.

3. Attainable temperature limit

Let us now turn to have an estimate of the equilibrium temperature attainable by the above two photon transition scheme. Because of the radiation pressure on the atom, the atomic velocity is damped, as if the atom were moving in a viscous medium. The velocity capture range Δv of such a process is obviously determined by the natural width Γ of the atomic excited state

$$k\Delta v \sim \Gamma, \tag{11}$$

where k is the wave number of the laser wave. On the other hand, by studying the competition between laser cooling and diffusion heating introduced by the random nature of spontaneous emission, one finds that for two-level atoms, the lowest temperature T_D that can be achieved by such a method is given by [14]

$$k_B T_D = \hbar \frac{\Gamma}{2}. \tag{12}$$

T_D is called the Doppler limit. For broad atomic lines, the energy width of the excited state $\hbar\Gamma$ is large compared with the recoil energy $\sim \frac{\hbar^2 k^2}{2m}$, and leads to the well known cooling limit for two-level atoms.

$$E_k \sim \hbar \frac{\Gamma}{4} >> E_r = \frac{\hbar^2 k^2}{2m} \tag{13}$$

The residual kinetic energy E_k is of the order of $\hbar\Gamma$ and it is large compared with the recoil energy The problem is to overcome this Doppler limit and to go well below T_D, and even approach the recoil limit T_R given by

$$k_b T_R = \frac{\hbar^2 k^2}{2m} \tag{14}$$

In 1988 this limit was overcome on laser cooled sodium atoms at low powers [15]. Much less work has been done to the problem of Doppler cooling with narrow atomic lines ($E_r \geq \hbar\Gamma$). Wallis and Ertmer [16] did a study of laser cooling using a narrow atomic line. They obtained a minimum residual kinetic energy of the order of $\hbar\Gamma$ well below the single line limit. The one $(g(\omega)_{1p})$ and two $(g(\omega)_{2p})$ photon Lorentzian lineshape function for natural broadening are given as

$$g(\omega)_{1p} = \frac{\Gamma}{\left[\Gamma^2 + (\omega - \nu)^2\right]}, \quad (15)$$

$$g(\omega)_{2p} = \frac{\Gamma}{\left[\Gamma^2 + (\omega + \omega_s l - \nu)^2\right]}. \quad (16)$$

The above two eqs. (15) and (16) indicate that the excited state spectrum for two-photon transition is comparatively narrowed due to the dynamic stark shift. For our two-photon case, we find that the recoil energy $E_r = \frac{\hbar^2(k_1 + k_2)^2}{2m} = 4\cdot\frac{\hbar^2 k^2}{2m}$ for $k_1 = k_2$, *i.e.* the two-photon recoil energy is already four times as large as the one-photon recoil energy. Further, coupled with the fact that for the two-photon case the width of the excited state is much narrow than that for the one-photon case $E_r \to E_k$. This can result in temperatures much lower than the Doppler limit imposed by the usual one-photon two-level theory.

We now turn to the problem of evaluating the equilibrium temperature in this new cooling scheme. We seek the value of the momentum diffusion constant D_p due to the quantum fluctuations of the force, which ultimately determines how long an atom stays in the trap. We then calculate the equilibrium temperature resulting from the competition between the cooling described above and the heating from momentum diffusion.

$$k_B T = \frac{D_p}{\alpha} \quad (17)$$

where α is the friction coefficient and is approximately for one-photon case given by [14]

$$\alpha \sim -\frac{\hbar k^2 \Delta}{\Gamma} \quad (18)$$

For small excitations of the atom and in particularly for the standing wave case, the diffusion constant is given as [13]

$$D_p = \left(\frac{\hbar^2 \Gamma p}{4}\right)\left(k^2 + \alpha^2 + \beta^2 + 4\alpha^2 \frac{\Delta^2}{\Gamma^2} p^3\right) \quad (19)$$

with $g = 2g\cos(k.x)$, $\beta = 0$, $\alpha = -k\tan(k.x)$ for a standing wave case. For the two-photon case, we need to replace k by $2k$, Δ by Δ'. The equilibrium temperatures for one and two-photon case we calculated as

$$k_B T_{1p} = \frac{\hbar \Gamma^2 p_0}{} \times \left[1 + 256 \frac{\Delta}{\Gamma^2} p_0^3 \sin^2(kx)\cos^6(kx)\right], \quad (20)$$

$$k_B T_{2p} = \left(\frac{\hbar \Gamma^2 p_0'}{\Delta'}\right. \cdot \left[1 + 256 \frac{\Delta'^2}{\Gamma^2} p_0'^3 \sin^2(2kx)\cos^6(2kx)\right], \quad (21)$$

where $p_0' = \frac{2|g_0|^2}{\left(\Delta'^2 + \frac{\Gamma^2}{4}\right)} - \left(1 + \frac{4\Delta'^2}{\Gamma^2}\right)$

and $p_0 = \frac{2|g_0|^2}{\Delta^2 + \frac{\Gamma^2}{4}} - \left(1 + \frac{4\Delta^2}{\Gamma^2}\right)$.

An inspection of eqs. (20) and (21) leads to the important conclusion that at a given detuning, increasing the power increases the temperature for the one-photon case indefinitely but for the two-photon case, it initially increases and then decreases. Moreover, at a given power, using the two-photon scheme, much lower temperatures are attainable.

4. Conclusions

In conclusion, we have studied in this paper, a new laser-cooling mechanism that is based on two-photon transition. This work demonstrates that two-photon interaction with a radiation field significantly enhances the radiation pressure force experienced by the atom. Further, the fact that for two-photon case, the width of the excited state is much narrow than that for the one-photon case, temperatures much lower than the one-photon limit is anticipated. In addition, results indicate that laser power in two-photon case is now no longer a limitation in attaining low temperatures. These results should stimulate additional experimental and theoretical research in order to establish their potential for laser cooling.

Acknowledgments

This work was done partially within the framework of the Associateship Scheme of the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy. I also acknowledge financial support from the University Grants Commission, New Delhi vide Project No. F. 15-1/2001 (SR-I).

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